Topic 4: Transfer function models and digital filters

1. Transfer function models in s domain
   1.1. Pole zero representation
   1.2. Rational representation
   1.3. Partial fraction representation

1. Transformation between representations

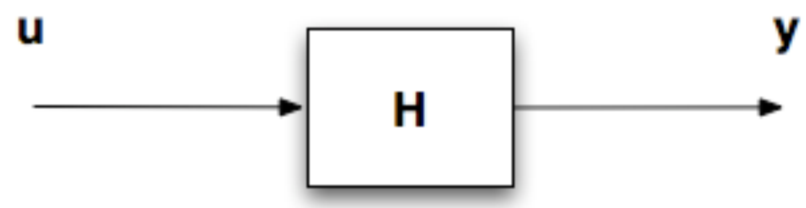
2. Modelling a system

3. Filtering data
   3.1. discretizing a model
   3.2. setting filter properties

4. IFO/Temperature example

Overview

- The general scheme: input, output and a transfer function



- Aim of this topic:
  - How to model the transfer function H in continuous domain, H = H(s)
  - How to discretize our model H(s) -> H(z)
  - How to filter data with H(z)
  - How to define H(z) from filter properties

Tools used here

Tools used here

1. Continuous domain

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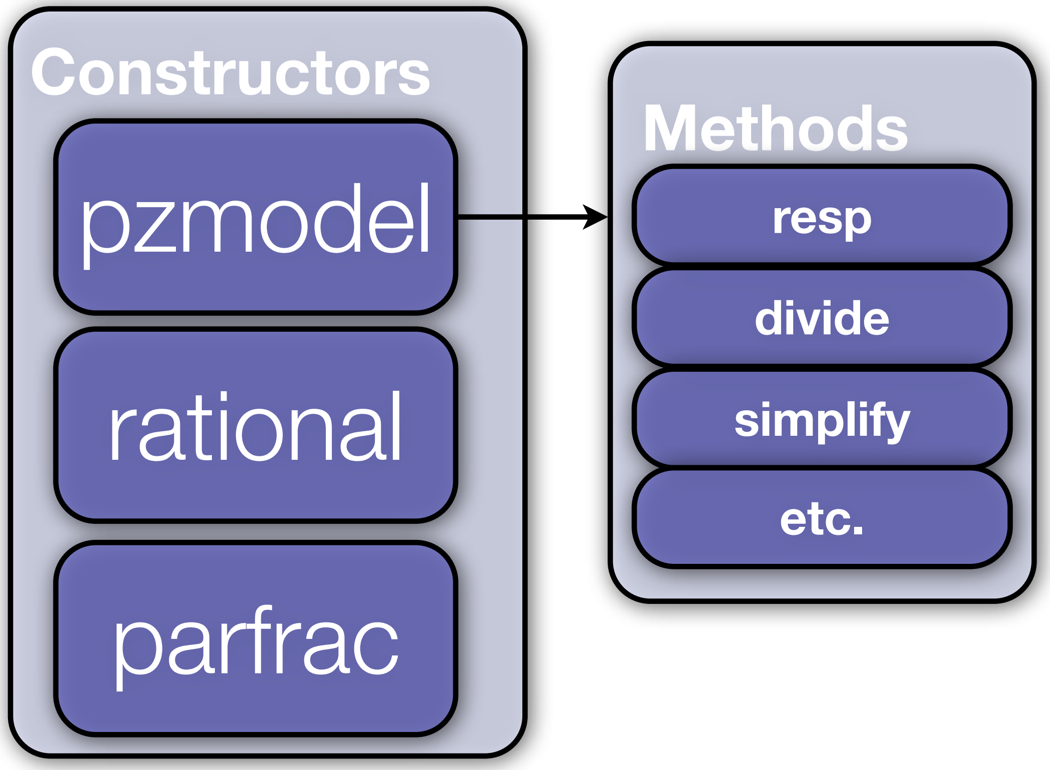
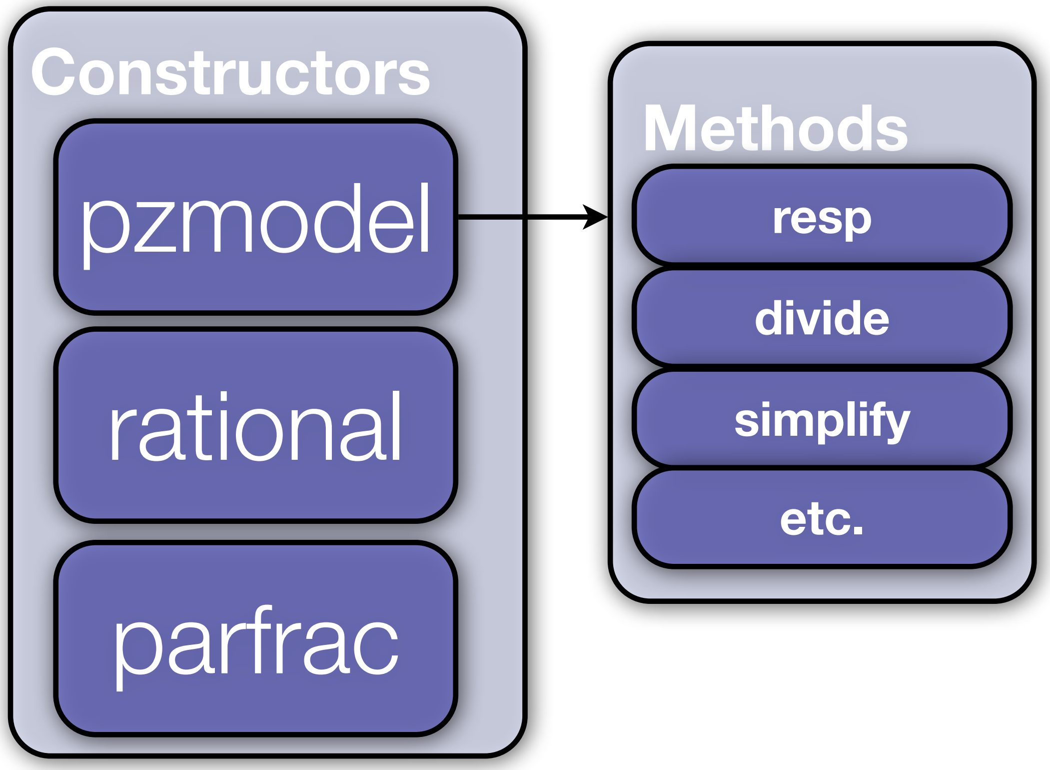


Tools used here

1. Continuous domain



2. Discrete domain

Tools used here

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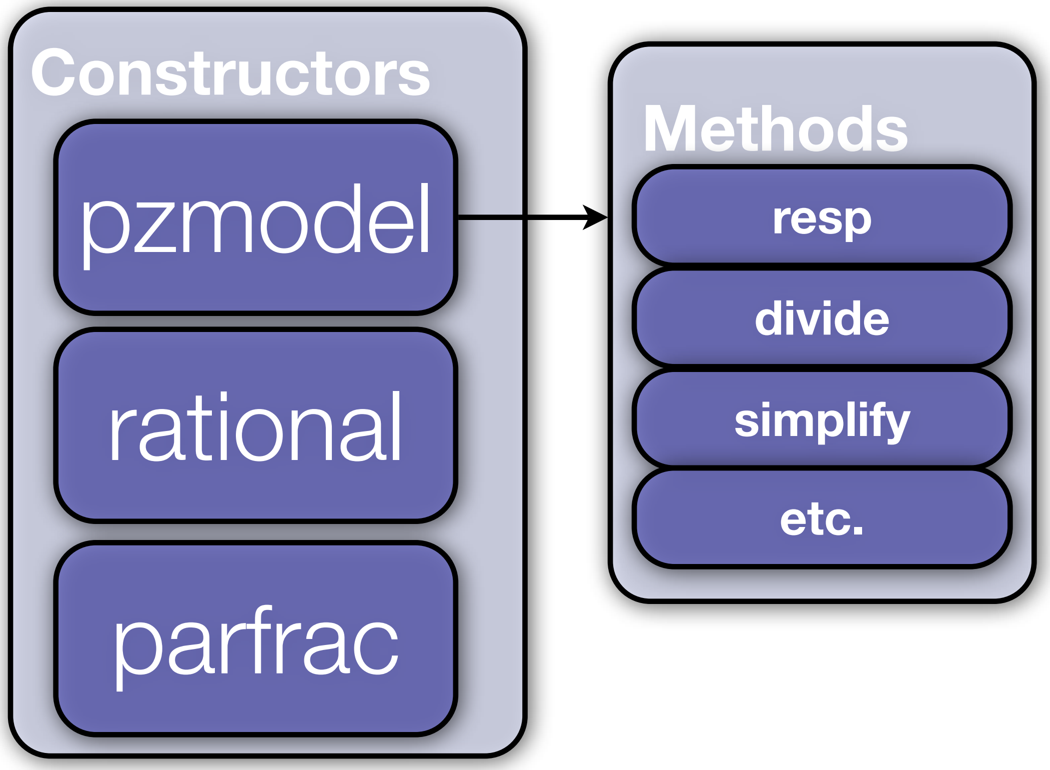
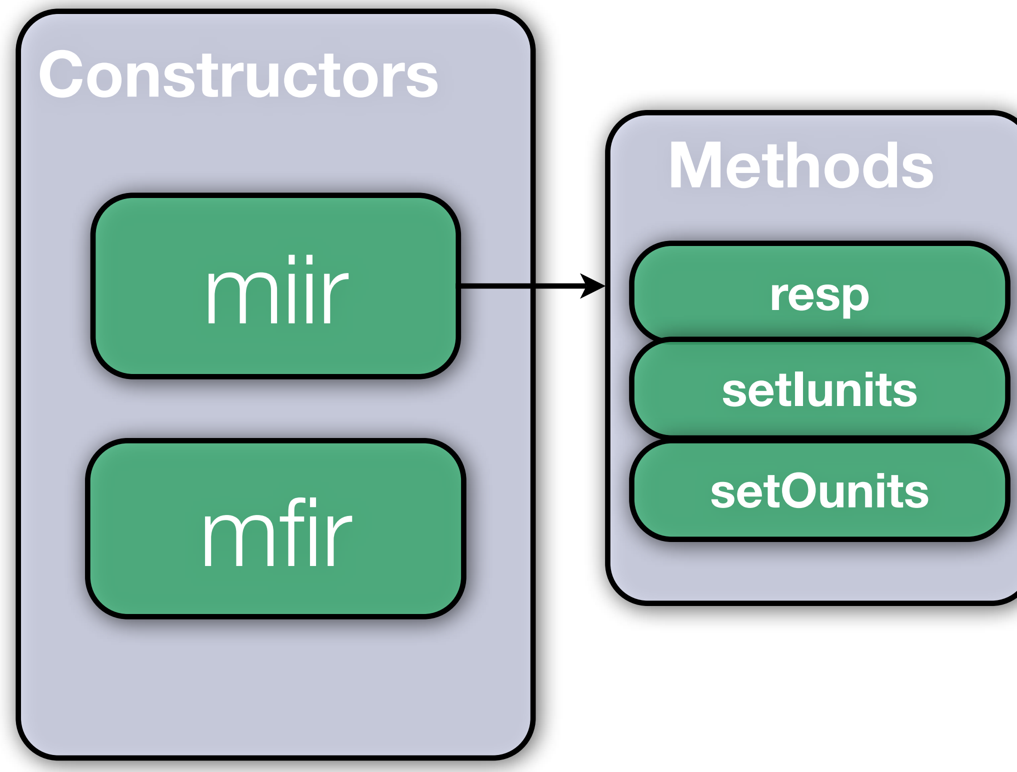


2. Discrete domain



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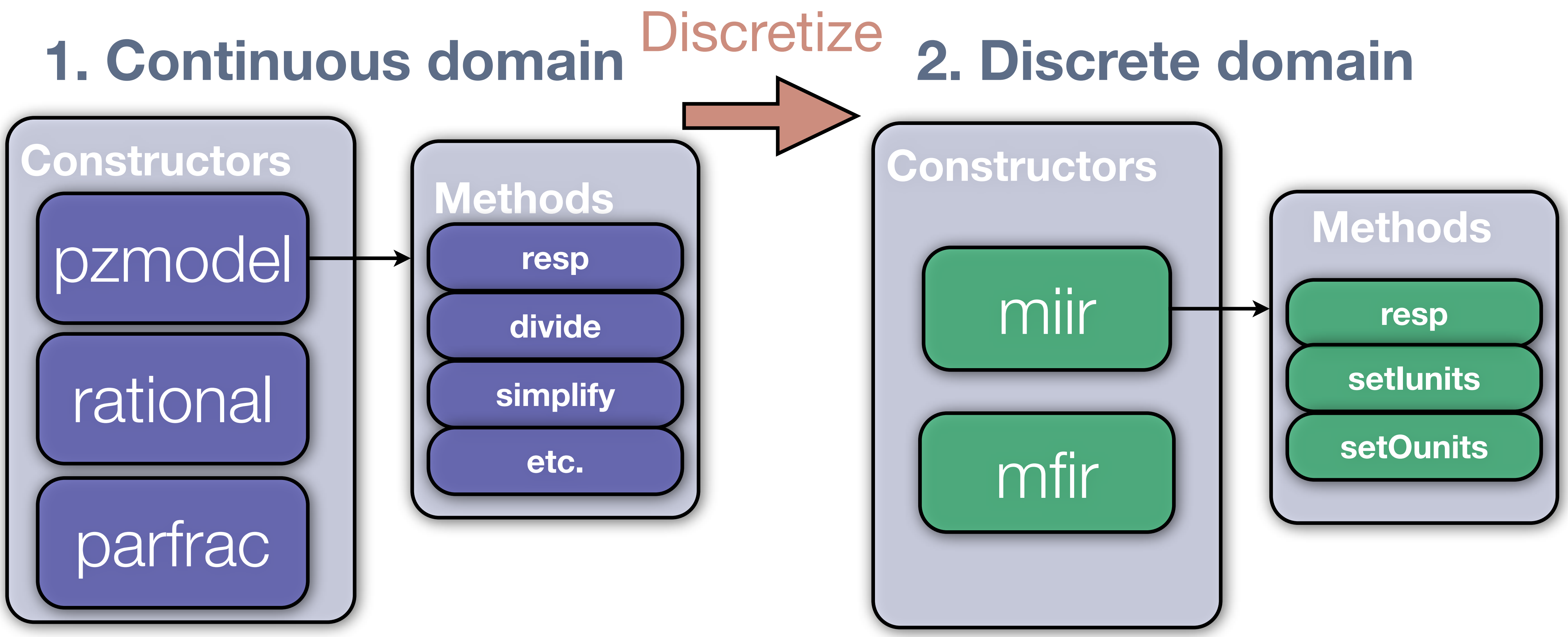

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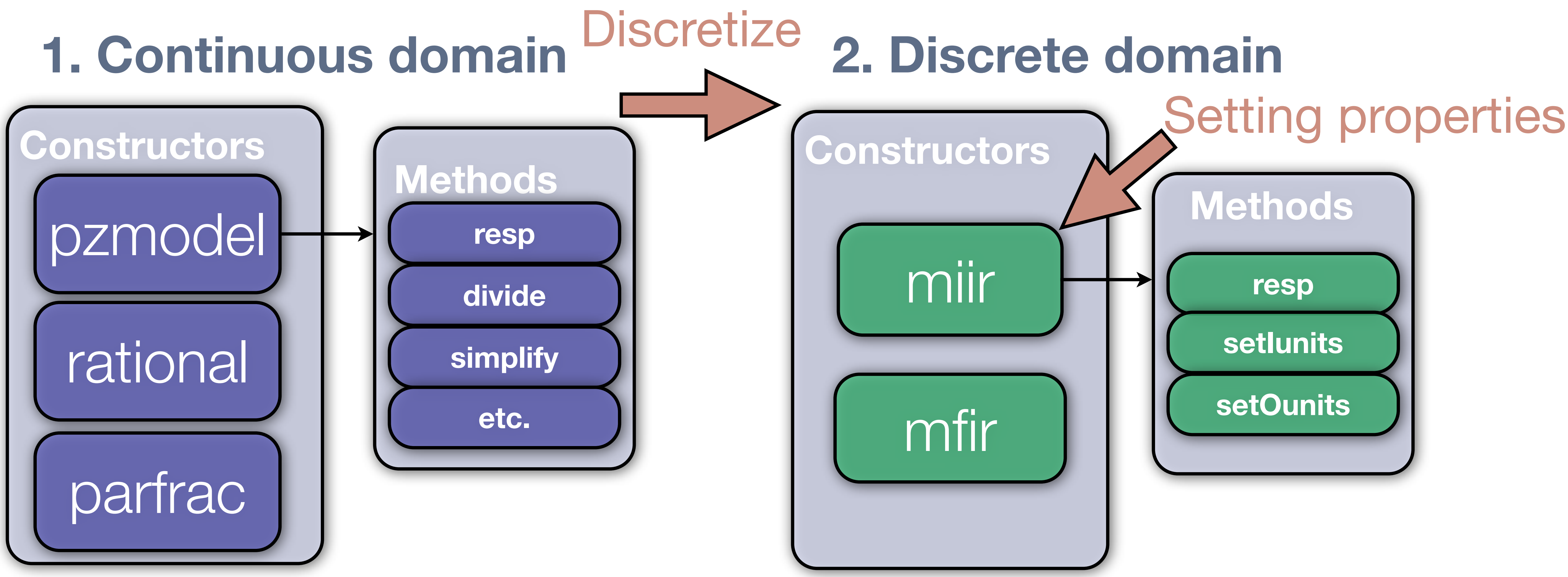

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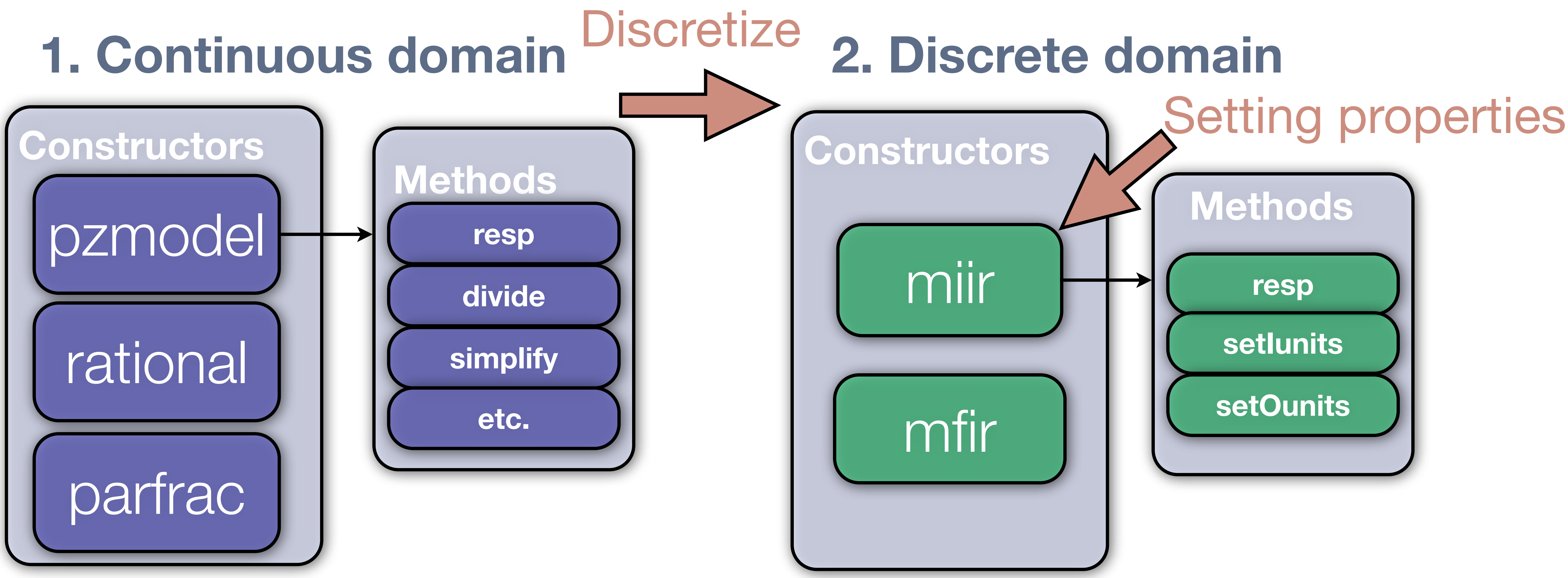
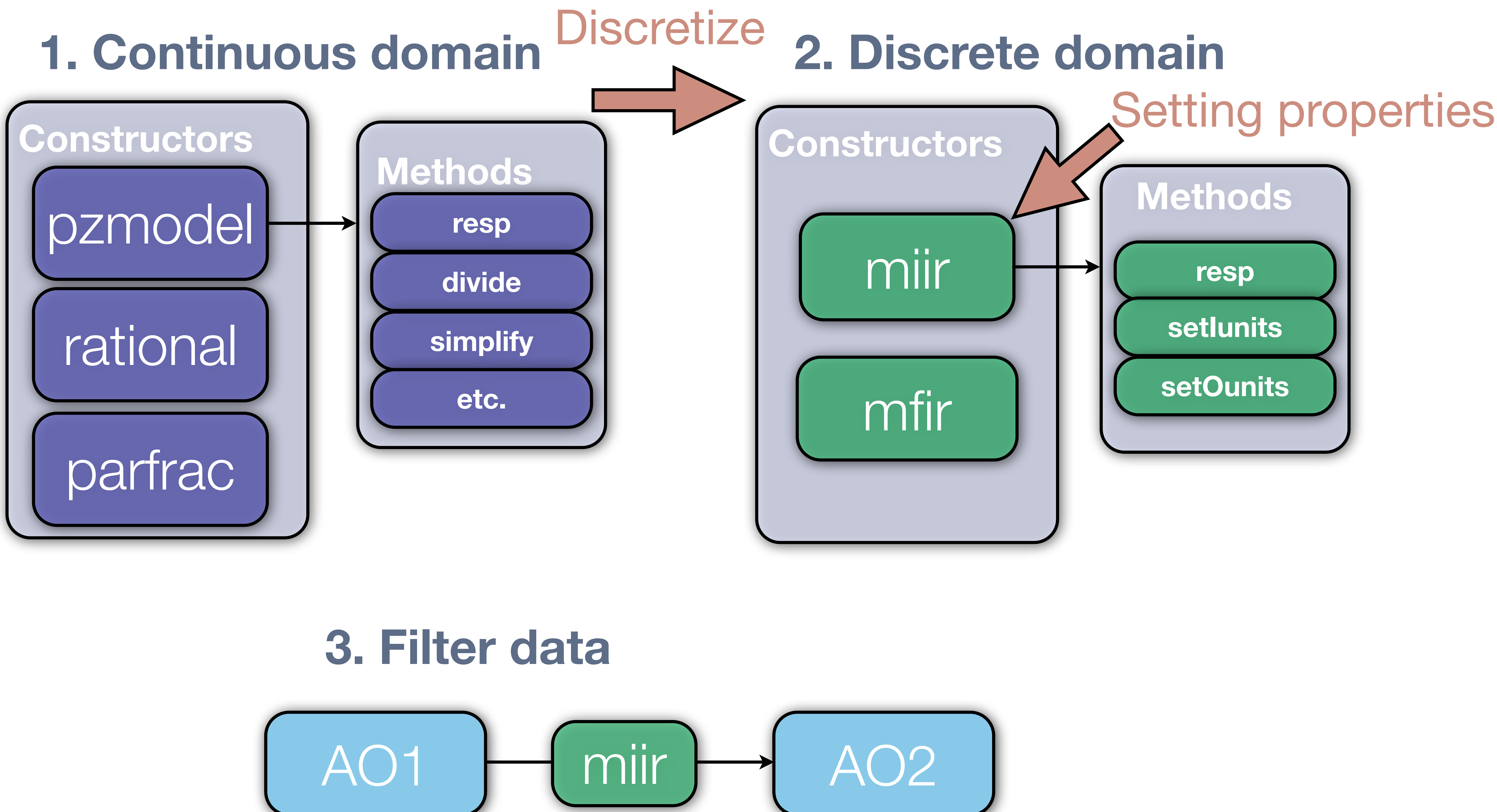


3. Filter data

Tools used here



1.1 Pole zero model

- A pole zero model is defined by:
  - Gain, poles, zeros, delay

$$H(s) = G\frac{(s-z_1)(s-z_2)\ldots(s-z_n)}{(s-p_1)(s-p_2)\ldots(s-p_m)}\,e^{-i\omega\tau}$$

- LTPDA constructor: PZMODEL
  - PZMODELs can be multiplied and divided
  - Delay is added or subtracted in such a case
  - Can read LISO files

About poles (and zeros) notation

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- Simple pole: f = 1 Hz

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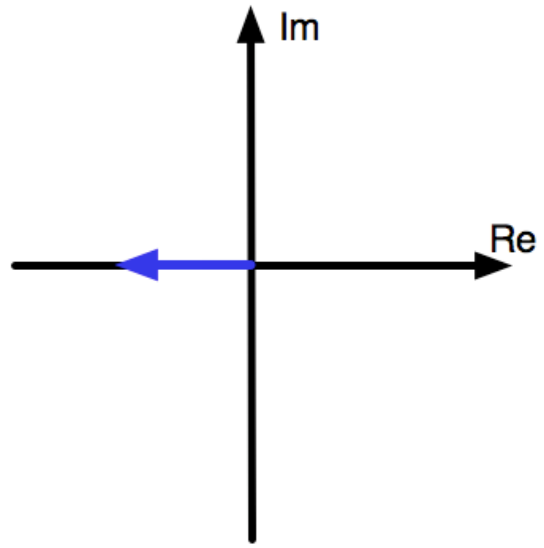
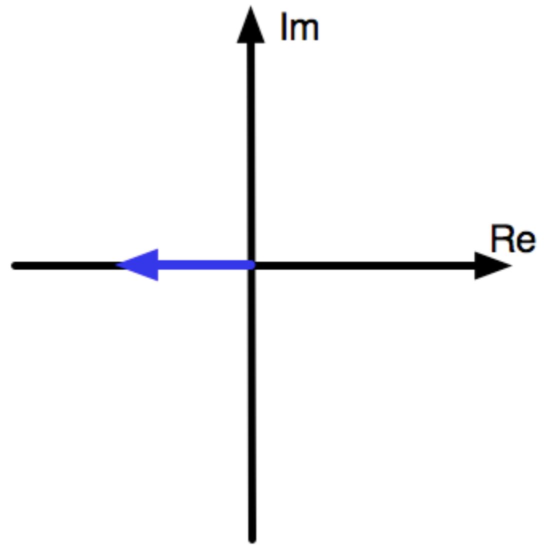


About poles (and zeros) notation

- Simple pole: f = 1 Hz
- Pole pairs: (f = 1 Hz, Q)



About poles (and zeros) notation

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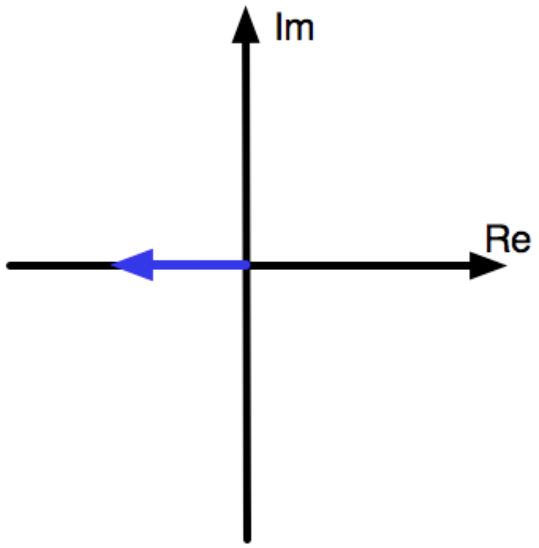
  Q > 0.5
  (underdamped)



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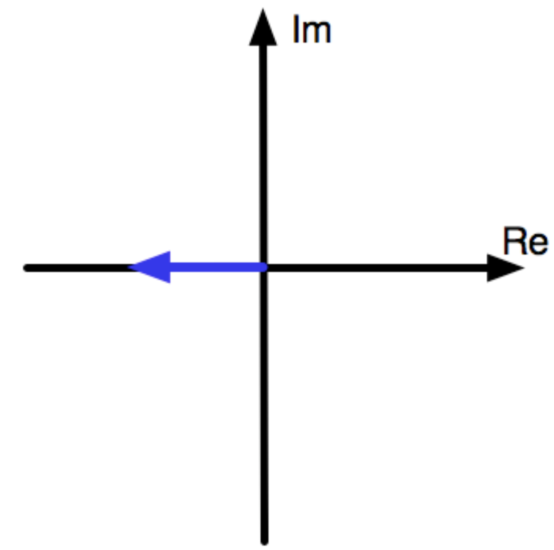

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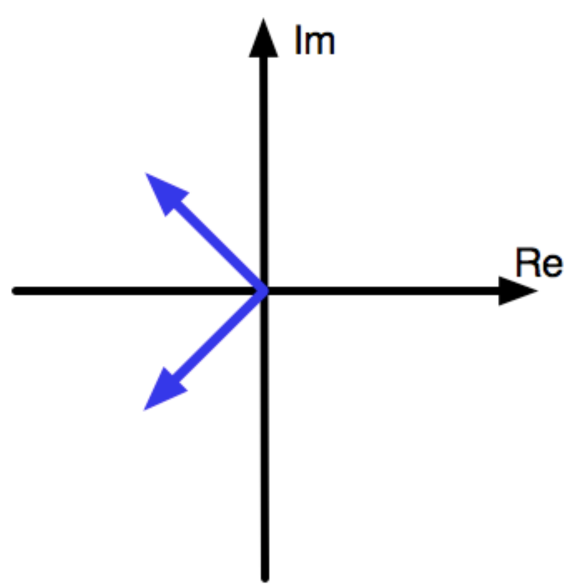

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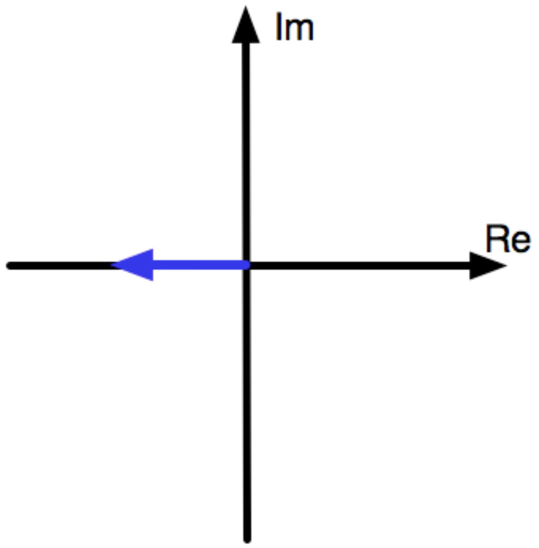

- Pole pairs: (f = 1 Hz, Q)

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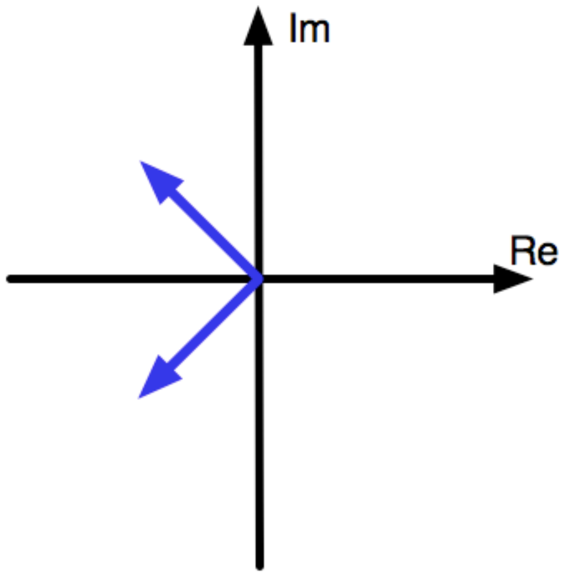
Q = 0.5
(critically damped)



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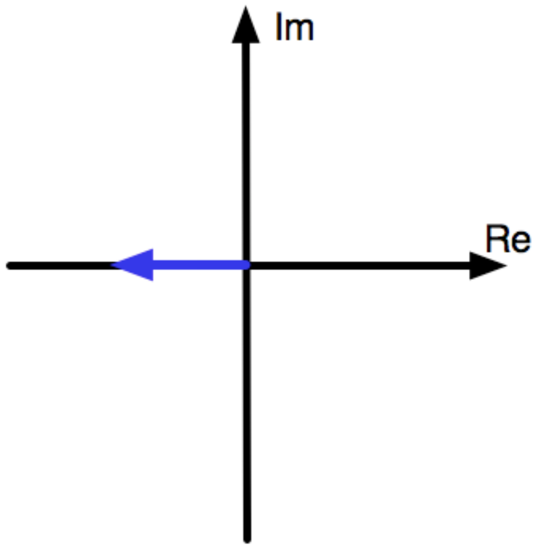

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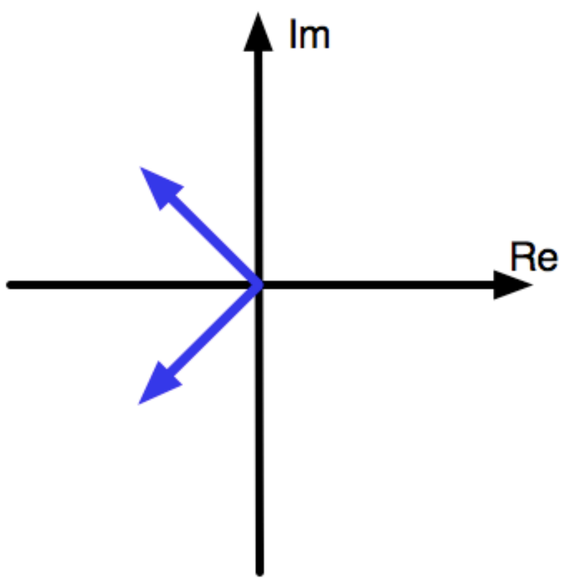

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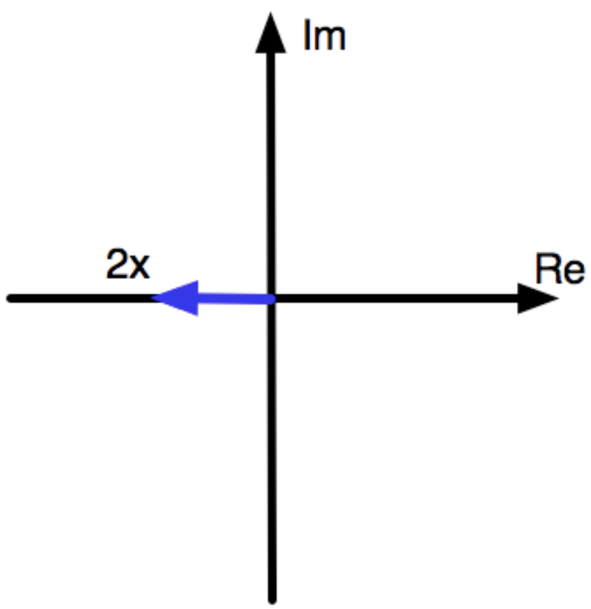

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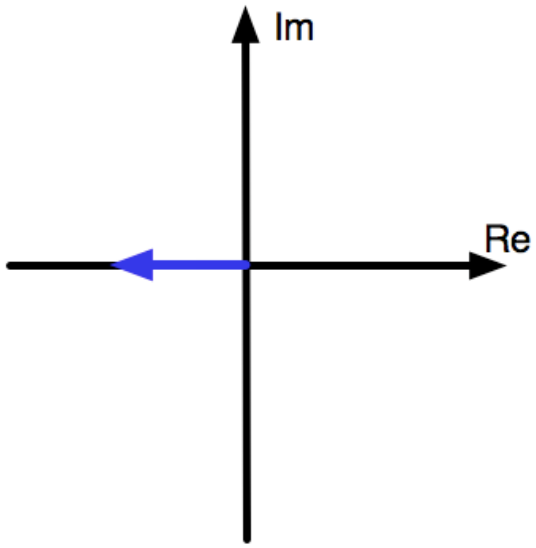

- Pole pairs: (f = 1 Hz, Q)

Q > 0.5
(underdamped)

Q = 0.5
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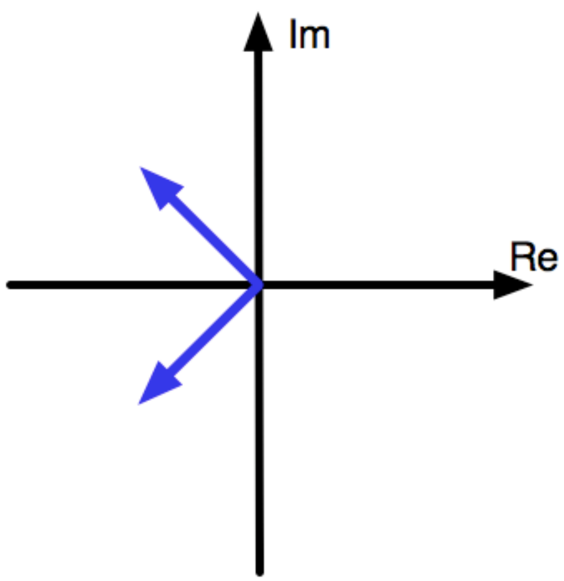
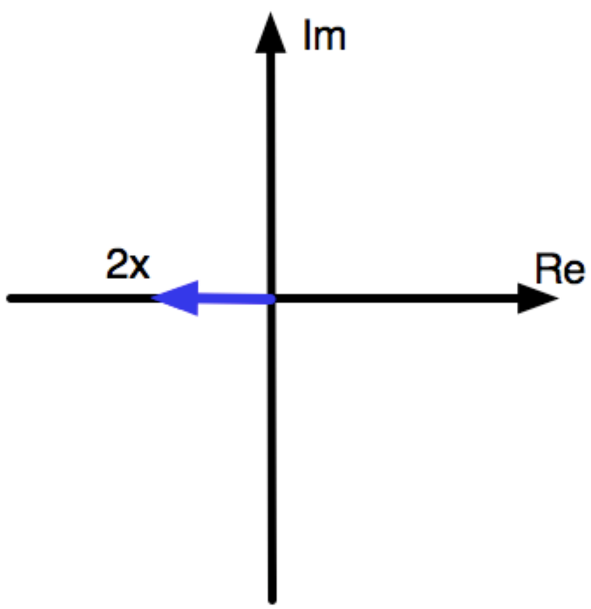
Q < 0.5
(overdamped)





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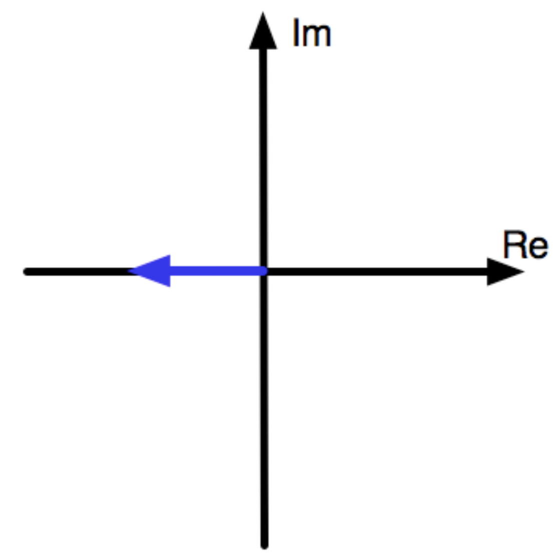

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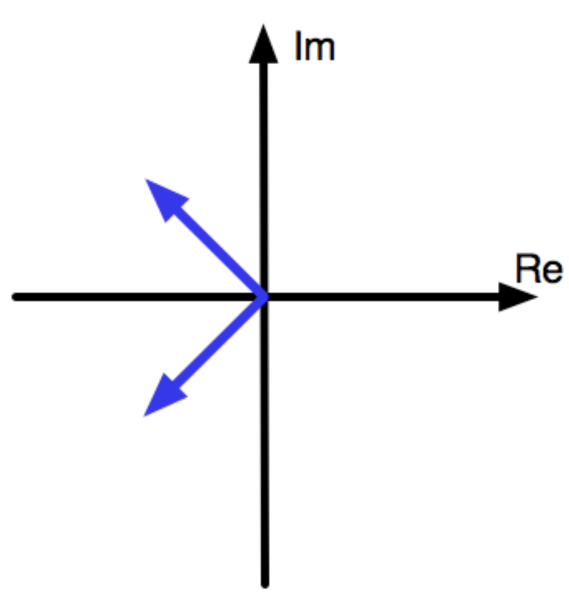

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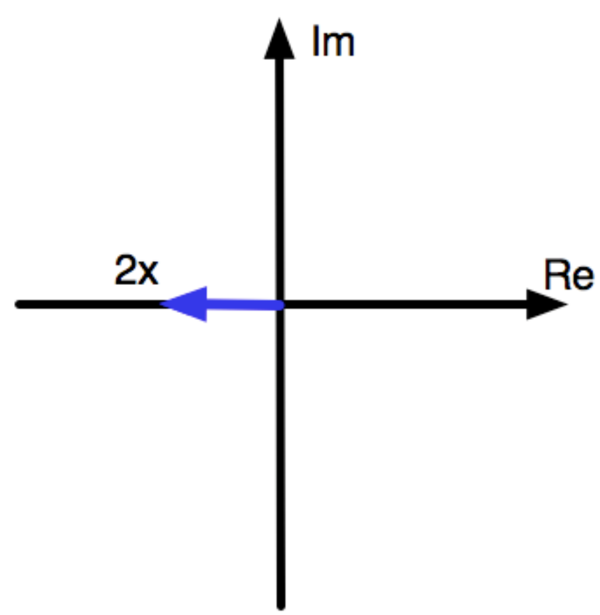

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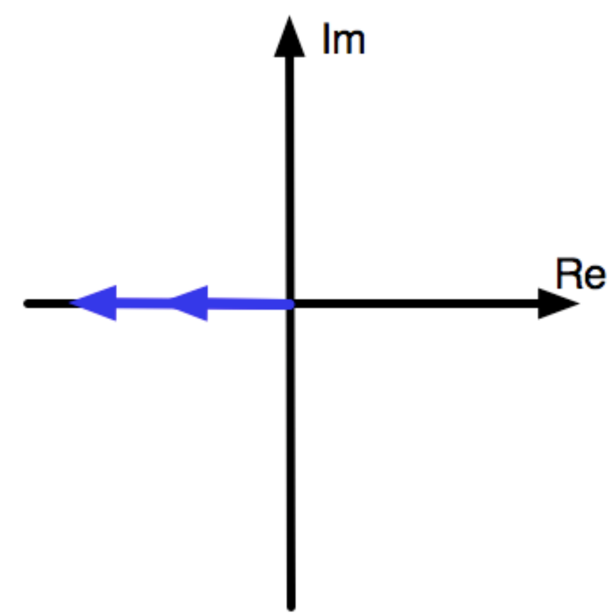
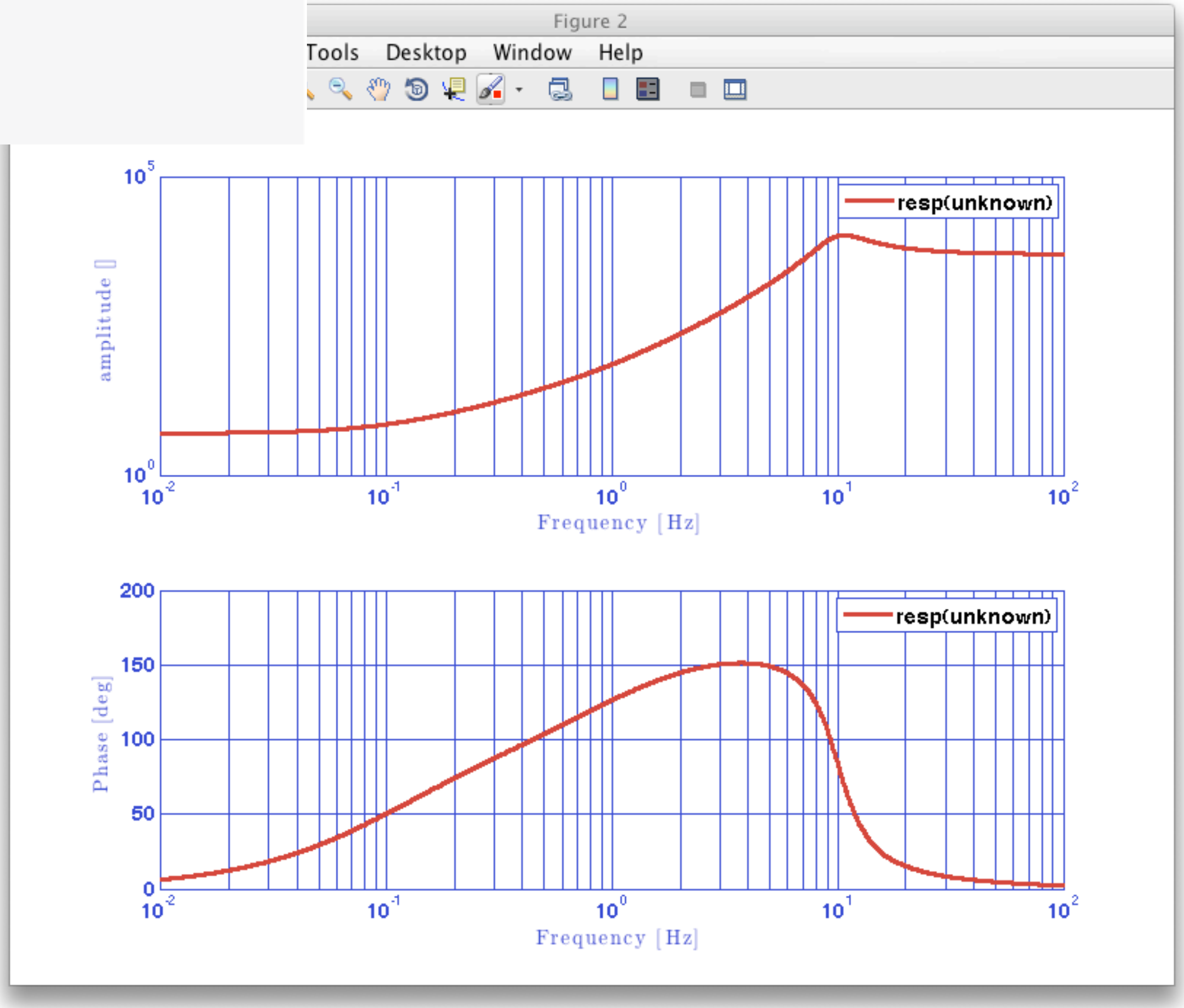


1.1 Pole zero model

- Working example: Compute pole zero response
  - Topic 4 > Create transfer func... > Create pole zero model

Key	Value
GAIN	5
POLES	(f = 1 Hz, Q = 2)
ZEROS	(f = 1 Hz), (f = 0.1 Hz)



1.2 Rational model

- A rational model is defined by:
  - Num. and den. coefficients

$$H(s) = \frac{a_1\, s^m + a_2 s^{m-1} + \ldots + a_{m+1}}{b_1\, s^n + b_2\, s^{n-1} + \ldots + b_{n+1}}$$

- LTPDA constructor: RATIONAL
  - RATIONALs can NOT be multiplied and divided

- Working example: Compute rational response
  - Topic 4 > Create transfer func... > Create rational model

1.3 Partial fraction model

- A partial fraction model is defined by:
  - Poles, residues and direct terms

$$H(s) = K(s) + \sum_{i=1}^{N} \frac{R_i}{s - p_i}$$

- LTPDA constructor: PARFRAC
  - PARFRACs can NOT be multiplied and divided

- Working example: Compute par. frac. response
  - Topic 4 > Create transfer func... > Create par. frac. model

1.4 Transforming models

- Only rational <-> pzmodel translation is implemented in v2.0
  - Works inserting object into constructor, e.g. rat = rational(pzm)

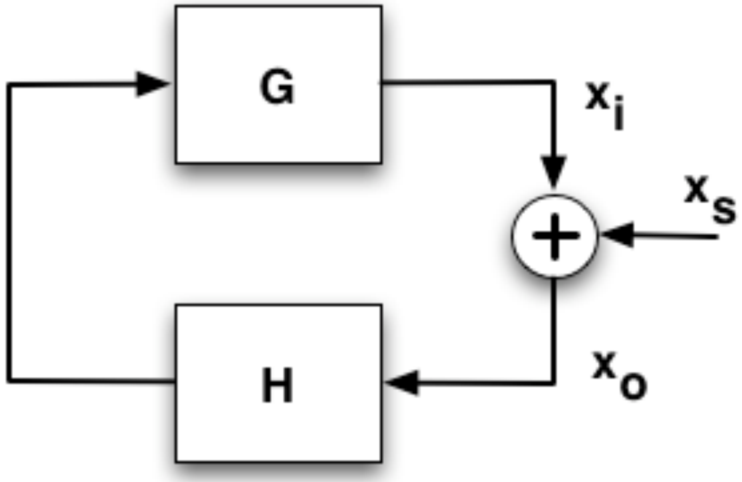
	Pole/Zero	Rational	Partial Fraction
Pole/Zero		√	X
Rational	√		X
Partial Fraction	X	X	

- Working example: pzmodel -> rational -> pzmodel
  - Topic 4 > Transforming models between representations

2. Modelling a system

- Modelling a closed loop system with pzmodels
  - Basic pzmodel operations

- Our system:



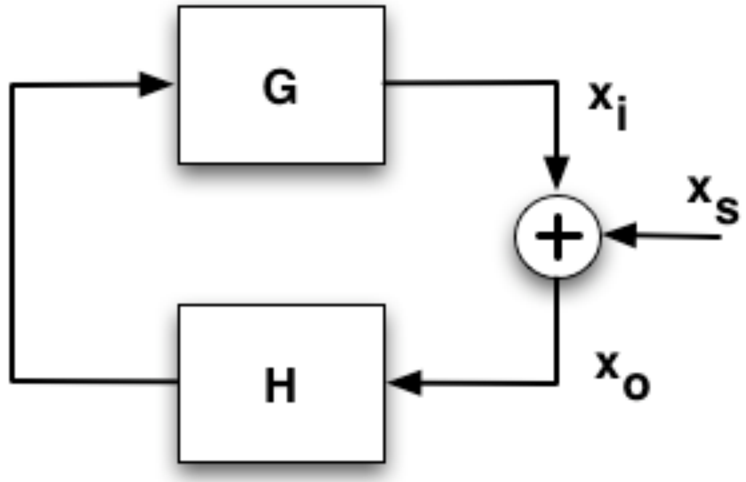
$$\frac{x_o}{x_s} = \mathrm{CLG} = \frac{1}{1 - \mathrm{OLG}} = \frac{1}{1 - \mathrm{H} \cdot \mathrm{G}}$$

- Stating the problem
  - Assuming OLG and H known, determine H and CLG

2. Modelling a system

- Step-by-step

  1. G = OLG/H
     (G is a pzmodel)
  2. Operate on G: setName, simplify ...
  3. CLG = 1/(1-OLG)
     (CLG is NOT a pzmodel)
  4. Repeat loading H with delay



$$\frac{x_o}{x_s} = \mathrm{CLG} = \frac{1}{1 - \mathrm{OLG}} = \frac{1}{1 - \mathrm{H} \cdot \mathrm{G}}$$

- Working example: Modelling a system
  - Topic 4 > Modelling a system

3. Entering the discrete domain

- The LTPDA toolbox allows you to build digital filters...
  - Discretizing your model
    - Example: find the filters for H,G, OLG in our closed loop
  - Defining filter properties
    - Example: Design a bandpass filter to evaluate power spectrum in a bandwidth

- Filter constructors in LTPDA
  - MIIR (IIR filters)

$$y[n] = \sum_{k=0}^{N} b[k]\, x[n-k] - \sum_{k=1}^{M} a[k]\, y[n-k]$$

  - MFIR (FIR filters)

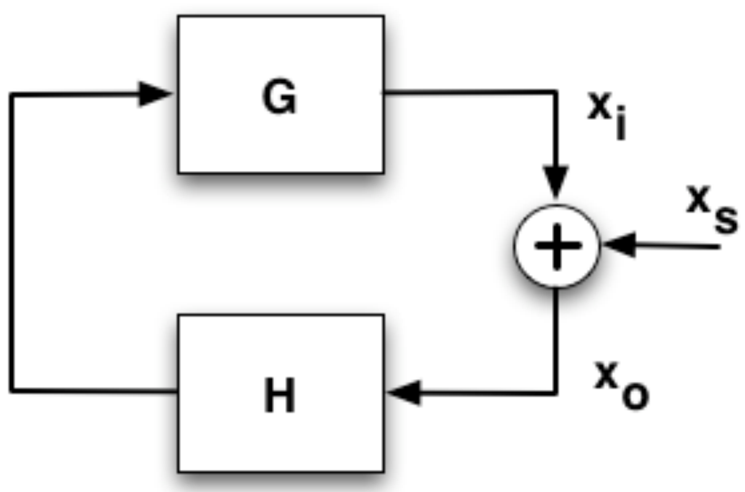
$$y[n] = \sum_{k=0}^{M} b[k]\, x[n-k]$$

3.1 By discretizing a transfer function

- Syntax: insert pzmodel into constructor

```
Gd = miir(G,plist('fs',10));
```

- miir → Constructor
- G → pzmodel obj.
- Gd → filter obj.



- Step-by-step
  1. Discretize G,H,OLG
  2. Compare continuous and digital response
  3. Get filter coefficients
  4. Delay is NOT used in the discretization!!

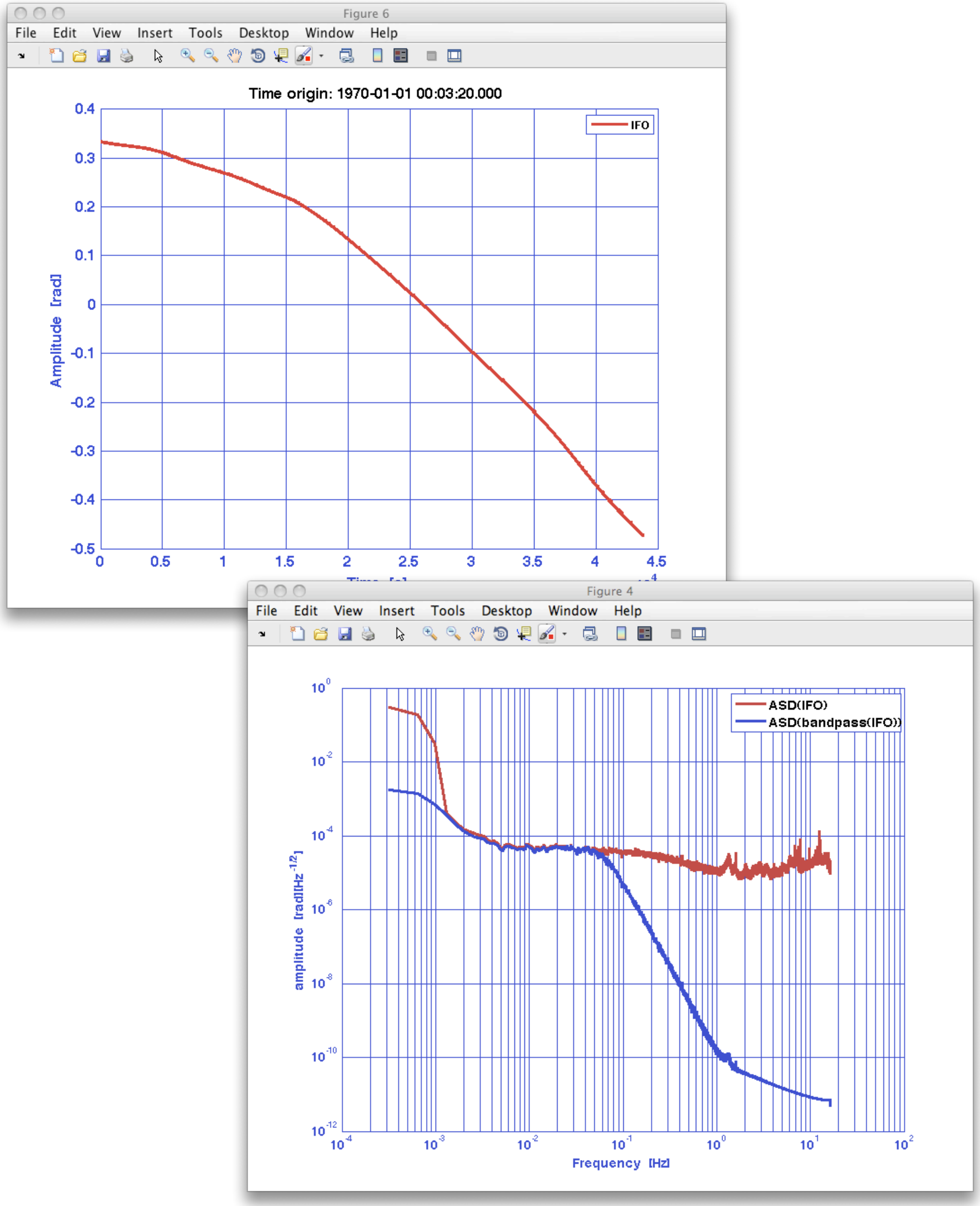
- Working example: Get filters for closed loop pzmodels
  - Topic 4 > How to filter data > By discretizing...

3.2 By defining filter properties

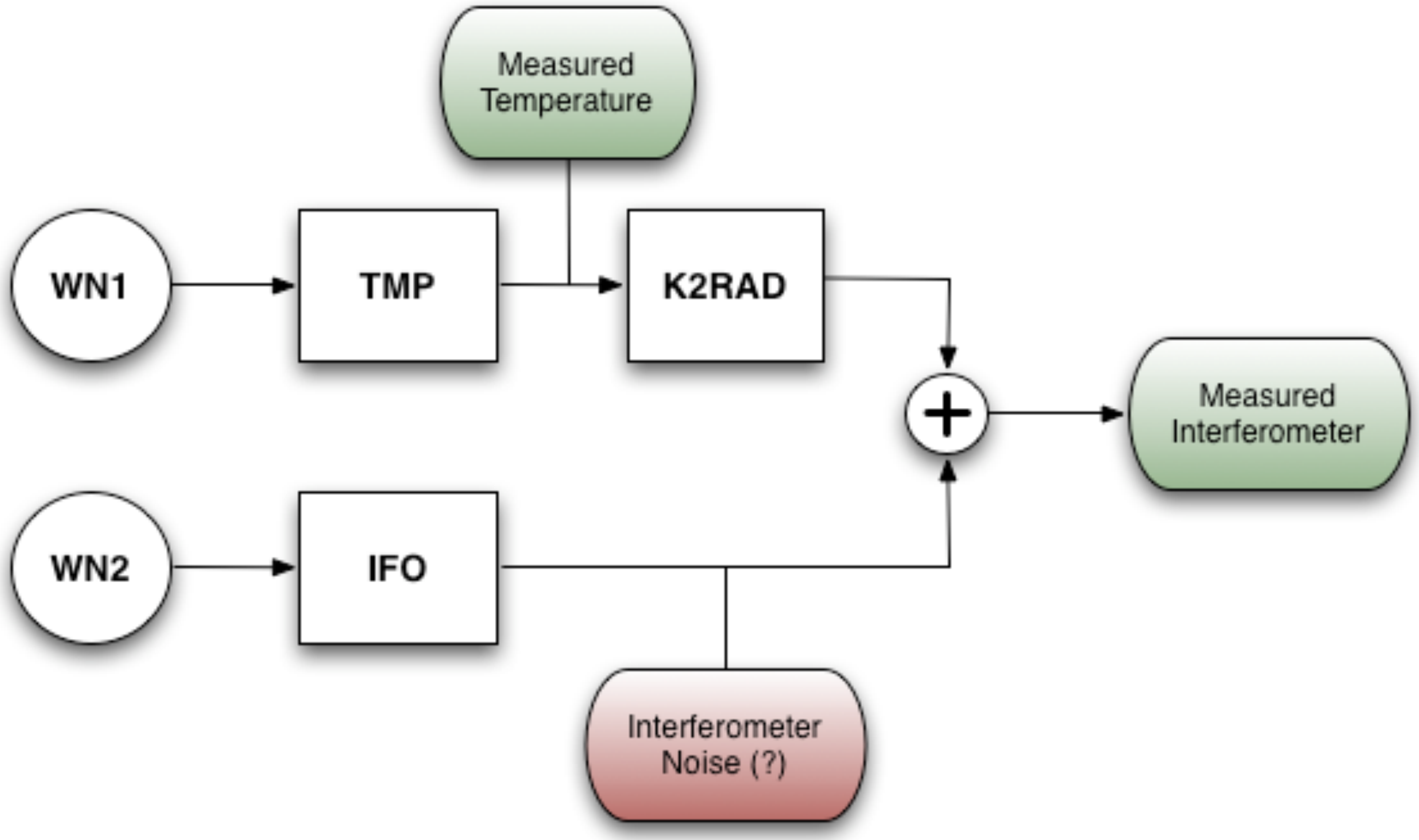
- Design a bandpass filter
  - Standard pre-processing step used in LTP lab
  - Alternative to detrending
- Syntax:

```
Gd = miir(plist('fs',32.47, 'order',...));
```

- Working example: Band pass



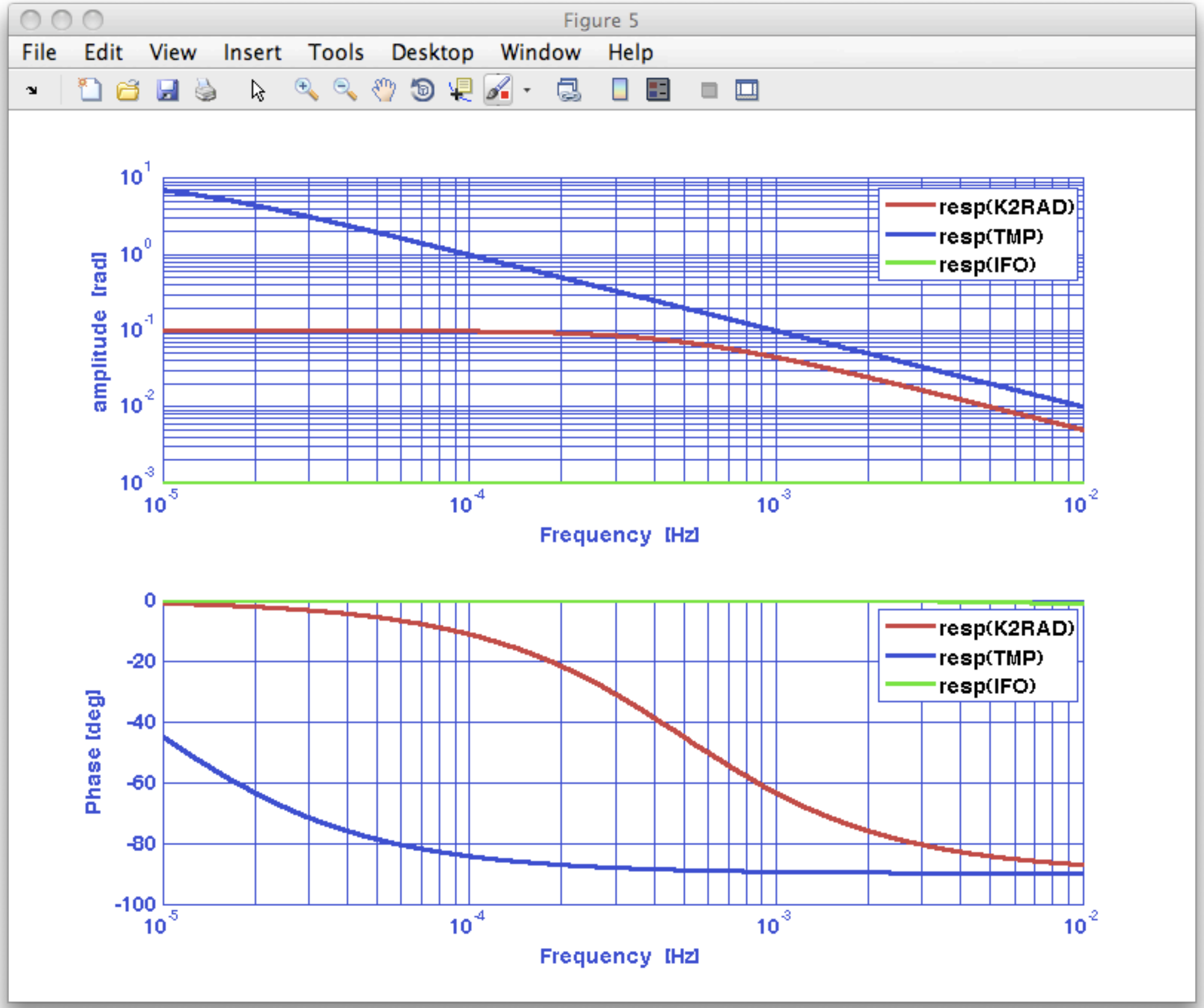
4. IFO/Temperature Example



- Aim: perform the analysis with a toy model
    - Create transfer function models: TMP, IFO,K2RAD
    - Discretize
    - Filter (white noise) data
    - Estimate transfer function (topic 3) with synthetic data

4. IFO/Temperature Example

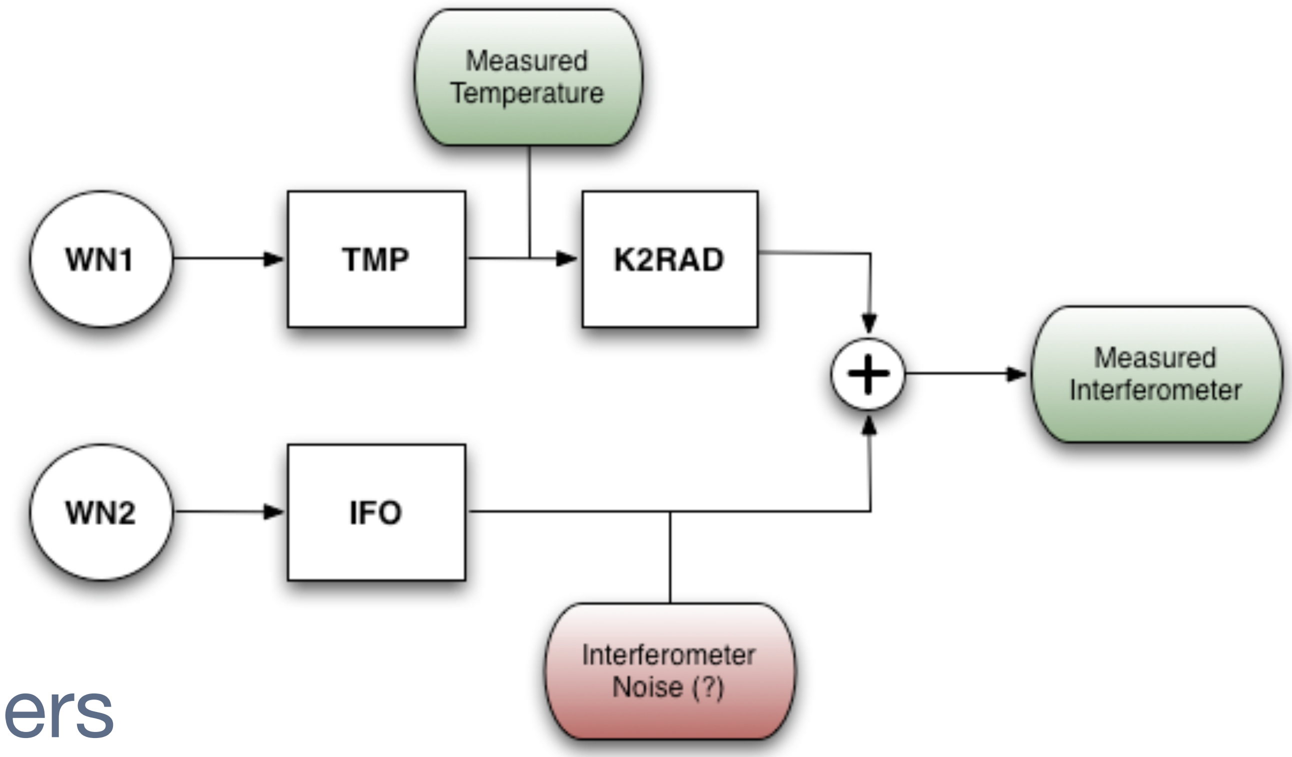
- Our toy models

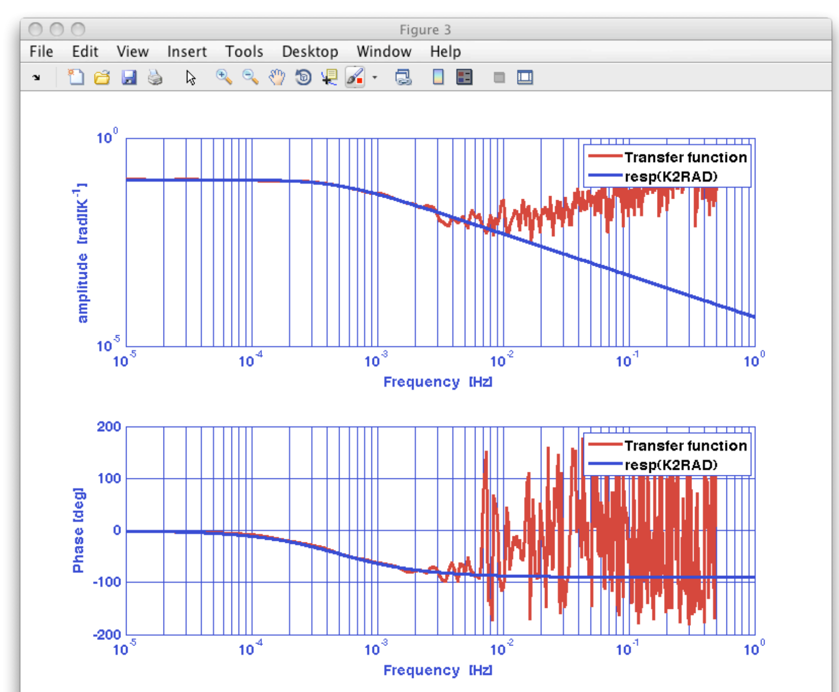


4. IFO/Temperature Example

- Step-by-step

1. Generate models: TMP, IFO, K2RAD
2. Discretize
3. Build two white noise time series
4. Filter with the digital filters
5. Estimate transfer function
6. Project temperature noise





- Working example: IFO/Temperature